Teaching Simple Structural Rigidity Theory in the Secondary Classroom

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Abstract

A unit of study teaching structural rigidity theory is developed for high school students of varied mathematical abilities. Students are led through the basics of building models of 2-D structures, understanding the concept of rigidity, and learning to predict the structural characteristics. This work is differentiated by dividing it into three levels of difficulty: Predicting using a counting algorithm, Developing an equation for prediction, and Proving the derived equation. An appendix is provided giving relevant National and Arizona academic standards.

Purpose

Beginning in the summer of 2007, this group was formed as part of the Math and Science Teaching Fellowship (MSTF) program at Arizona State University. The fellowship gave us the opportunity to work with university researchers and create curriculum for the high school based on the experience. The authors were assigned to work with the Center for Biological Physics (CBP) with Dr. Banu Ozkan, whose research interest is computational prediction of protein folding.

Much of this work is done at a level that is not appropriate for the high school classroom, but it was our task to identify elements of this work that *could* be introduced to younger students. One aspect of protein structure that seemed to meet this requirement was the study of structural functionality. Proteins contain both rigid and flexible regions that contribute to their ability to carry on their biological mission.

In a lecture by Dr. Michael Thorpe (CBP Director) on just this topic the authors were shown a model that demonstrated basic structural forms that were either rigid or flexible. This model was built with custom metal parts and fasteners, constructed in a university machine shop. The authors noted that the cost of using such models is out of reach for most secondary schools. The concept was appealing enough for the authors to seek an affordable alternative. Brainstorming ensued, and after some visits to toy stores, hardware stores, and storage closets at schools, the authors found that common popsicle sticks (often marketed to schools as "craft sticks") could be used. Not only were they more affordable, but craft sticks have been found to offer greater opportunities for discovery and varied investigations.

This document will provide information for building these models as well as discussion of the curriculum itself, including development of mathematical models by students.

Description of Equipment

The basic unit is the craft stick. These are wooden sticks, approximately 3/8” x 4 ½” with rounded ends. The authors found that the sticks could have 1/16” holes drilled near the end to facilitate fastening the sticks together. Some experimentation was done on the choice of connectors, beginning with paper clips (not effective). The authors found that simple cotter pins (also known as split pins), 1/16” x 1” in size. As elements of complexity were added to the lessons, the authors found that sticks of different length could be useful.

Teaching Rigidity with Modeling

The unit is organized to teach these principles in an inquiry setting, using pedagogy inspired by the Modeling Physics program. Students are presented with the equipment, given some basic information, and asked to construct models that exhibit both rigidity and flexibility. Through student discussion and guided challenges, students will develop the basic mathematical algorithms that can be used to predict flexibility of simple 2-D structures. In this way, students develop accurate and detailed models of structures. We also feel that this material can be differentiated so that student groups or individual students with different age or ability levels can benefit. To facilitate this, we have divided the material into three progressively challenging levels.

Level One (for all students) How to begin:

I. Discussion – students view images of both natural and man-made structures (may be provided by teacher or can be an Internet search) and make observations – differences, similarities, and patterns seen in both.

   Teachers’ note: Teacher should direct discussion into patterns that are responsible for the durable or rigid structures in each image.

   Guidelines for appropriate observations: geometry of sub-structures, redundant components, and selection of materials.

II. Activity – Students are given sets of craft sticks and cotter pins to facilitate the hands-on component of the exploration of physical and mathematical characteristics of structures.

Teachers' note: Sets of craft sticks are prepared and provided to individual students or to appropriately sized groups of students. Pairs are ideal, three students will usually work; more than three students in a group usually is less effective. Basic guidelines should be given concerning obvious safety issues, not bending cotter pins (the pins will need to be reinserted multiple times, and don't survive multiple bending and straightening cycles).

A number of concepts and terms will be used in this unit. The teacher will need to provide definitions and limitations.

1. These structures are all limited to two dimensions, those of the students' tables.

2. In order to create definitions of structures, it is important to describe how objects are allowed to move. A single craft stick can move three different ways: horizontally, vertically, and may rotate about its center. All these motions are confined to the two dimensions described above. These allowed motions are called "degrees of freedom" (DOF). Note that two *unattached* sticks would have a total of six degrees of freedom, three would have nine degrees of freedom and so forth.

   Teachers' note: Please refer to the companion PowerPoint presentation for an explanation complete with graphics. While many different structures can be created, it is important to limit early examples to simple structures with no more than two sticks at any vertex.

3. When sticks are joined with cotter pins, certain constraints arise. In the simplest example, two sticks are joined with one cotter pin. To do this, one stick must be moved to the other so that the holes at their ends align. In doing this motion, the stick being moved is forced to move in the horizontal and vertical direction until the coordinates of its end are the same as those of the other stick. Thus the moved stick "gives up" two degrees of freedom. The resulting structure then has four degrees of freedom – the entire assembly may be moved vertically or horizontally, rotated, *or using the pin as a hinge, the structure now has an "internal floppy mode",* where the angle between the two sticks may be changed.

Teachers' notes:

• *Each stick added to a system by attaching with a new or existing cotter pin surrenders two of its degrees of freedom in the attachment process. Teachers should determine whether to use the term "constraint" or simply refer to "lost degrees of freedom" (LDF).*

III. Teaching the "counting" algorithm for DOF for a structure

1. Using one stick (or by drawing a line or stick on the board), review the three degrees of freedom that it has.

2. Show that two unattached sticks have six ($3 + 3 = 6$) DOF.

3. Show that moving a stick to align its hole with that of another stick in order to join them requires using two of the degrees of freedom that it has. Once they are joined, those degrees of freedom are lost. Thus this structure now has four degrees of freedom ($2 \times 3 + (-2) = 4$)

4. The next example should be three sticks joined using two pins. Point at each vertex and ask students how many DOF are lost by making the connection and write this number at the vertex. Students may also build this structure and verify that there are five DOF ($3 \times 3 + 2(-2) = 5$).

5. Now join the three sticks to form a triangle. As in number 4 above, prompt students to identify the DOF lost and calculate the structure's total DOF, which is three ($3 \times 3 + 3(-2) = 3$)

6. Join four sticks with four pins to make a square. Have students use the counting techniques to determine the DOF ($4 \times 3 + 4(-2) = 4$) and verify this answer with the actual model.

   - *With each structure, students should determine whether the structure is rigid, flexible, or some combination of rigid and flexible elements.*
   - *Describing even these simple examples verbally quickly becomes complex. As structural elements are added, this complexity grows. Please refer to the companion PowerPoint presentation for an explanation complete with graphics.*

IV. Activity

Each group of students should now be asked to create original structures with their sticks and pins, physically determine the DOF for their structure, and verify this using the counting algorithm. This should be done as a modeling activity where groups show their work on whiteboards or using other presentation technology to present their work to the class. Class discussion should center on understanding how different structures create rigid or flexible elements.

Teachers' notes:

- *Students should be led to observe that structures with DOF = 3 are completely rigid, and that structures with DOF > 3 have internal "floppy" modes equal to the structure's DOF minus 3.*

• *It is important to give students rules regarding redundancy (although the term need not be introduced as vocabulary). If these rules are not followed, the counting algorithm will give a false answer.*

✓ *Two sticks that are not attached to each other may not cross.*
✓ *If there is a stick you can take out and the structure is still rigid, that was a redundant stick.*
✓ *If a structure is already rigid, you cannot add another stick to it.*

Level Two – Deriving an algebraic equation to predict rigidity

This level is intended for students with intermediate mathematic and cognitive skills. Begin by posing these questions to students:

> *"Suppose a structure contains hundreds of sticks and pins. Will the counting algorithm be easy to use?"*
> *"What do scientists create, using patterns, so that they can predict characteristics of complex systems?"*
>
> Suggested discussion idea:
> - "As scientists investigate a phenomenon, they not only understand it physically, they also try to develop a mathematical model of the phenomenon. Why do they try to do this?" *Scientists develop mathematical models in order to make quantitative predictions about structures, and engineers need mathematical models to choose materials and design structural elements that will not fail in normal use.*

Students should agree that complex structures can't be easily evaluated using the counting algorithm and that an equation or mathematical representation of some sort needs to be created.

The power of the modeling approach is that students are continuously reminded (indirectly) that such developments follow a pattern:

- No pattern exists until data is collected and organized so that it can be evaluated.
- Data collection should be done over a reasonable range or assortment of experimental samples
- Once data is collected, it needs to be evaluated so that a mathematical representation may be written.

It will be assumed for this level that students are competent using the counting algorithm.

> *Teachers' note: The following process is intended to do in two iterations. The first one will result in a false equation that will not work in all cases. The second will produce the correct result and is important as a "setup" for Level three.*

1. A discussion should be held to determine the variables to evaluate. The result of this discussion will yield pins, sticks, and DOF. As in all modeling instruction, the teacher leads student discussion to evaluate and discard other suggestions.

   Teachers' note: Use these symbols for the variables: P = # of pins, N = # of sticks, *and* F = DOF.

2. Before recording data, a table needs to be created. Here is an example:

P	N	F

3. It is suggested that the same series of simple structures used in Level one be used for this activity. Thus the first structure to evaluate and enter in the table will be the single stick.

P	N	F
0	1	3

4. The second structure would be two sticks joined with one pin.

P	N	F
0	1	3
1	2	4

5. Next is three sticks joined by two pins.

P	N	F
0	1	3
1	2	4
2	3	5

6. This is followed by the triangle (three sticks, three pins).

P	N	F
0	1	3
1	2	4
2	3	5
3	3	3

7. Finally, data is taken from the square structure (four pins, four sticks).

P	N	F
0	1	3
1	2	4
2	3	5
3	3	3
4	4	4

8. Now that data has been collected, analysis begins. Have the students make observations about the data and list any patterns that they see. Important observations are:
   - all numbers involved are integers
   - there seems to be a relationship between the number of pins, the number of sticks, and the DOF
   - pins and sticks are independent variables, DOF is dependent

9. The observation that all numbers involved are integers indicates that this pattern is linear, and students may be asked to create or may be shown that:

$$P + N \propto F$$

Students should be asked what is necessary to express this relationship as an equality; their mathematical experience should lead to the observation that constants of proportionality (some integer multiplier) are needed, yielding

$$\alpha P + \beta N = F$$

10. Led by the teacher, students should observe that, using the data, we should be able to solve for the two unknown variables (α and β) using a system of two equations based on two lines of data. The teacher may have groups do this as an independent activity, reporting their results after time has elapsed, or the teacher may lead the class by having some student select two lines of data from the table and solving the system of two equations. Example:

Eq. 1: $1\alpha + 2\beta = 4$	→	multiply by -2	$-2\alpha - 4\beta = -8$
Eq. 2: $2\alpha + 3\beta = 5$	→	add to this eq.	$2\alpha + 3\beta = 5$
		yields	$-\beta = -3$, or $\beta = 3$

Substituting in Eq. 1, we then find that $\alpha + 2(3) = 4$, or $\alpha = -2$
This give the equation

$$-2P + 3N = F$$

11. Ask students if they feel that the job is finished. Students should insist on testing this equation to verify that it provides the correct DOF for any structure. They will find that it works for every entry on the data table, so it should be suggested that students try to use a more complex structure to test this equation. One structure that could be used is simply to start with the triangle and add one stick that is connected to a vertex. Looking at the model, students should see that it has 4 DOF. Using the counting algorithm, this is verified. But using the new equation:

$$-2(3) + 3(4) = 6 \neq 4$$

    So this equation is not correct. The students should be allowed to confirm this mathematically with their models.

12. Record results from student models on the board, listing the correct number of DOF (using counting algorithm and model-bending) and the number of DOF calculated with the defective formula. Students will observe that the formula always over-reports the DOF, and that this error is always a multiple of two.

13. Discuss this error with students, point out that we are not deducting enough DOFs. Since our table shows pins, sticks, and DOF, and leads to this formula, we must change something in our approach. What can we change to "get rid of" DOF? How can we change our model-building? Point out that the number used to reduce DOF is the number of pins. Can we do something differently to the way we report the number of pins?

    Teachers' note: The only obvious conclusion is that we must increase the number of pins. Point out that there are holes on some sticks that do not have pins, while on some models all holes have pins. Are models with pinless holes the same as models with pins in all holes?

14. The same models are reevaluated, placing pins in all empty holes.

15. The first structure to evaluate and enter in the table will be the single stick. For convenience, all the following entries will appear on the table at the end of this procedure.

16. The second structure would be two sticks joined with one pin.

17. Next is three sticks joined by two pins.

18. This is followed by the triangle (three sticks, three pins).

19. Data is next taken from the square structure (four pins, four sticks).

20. The triangle with one additional stick is added to the table. This is important because this is the model that does not work with the old formula.

21. Have students discuss whether it would be prudent to create a more complex model to use in finding the formula. A suggested model is a square with two sticks added on top to form a triangle. This model looks like a house drawn by a small child. The following table will result from these models:

P	N	F
0	1	3
1	2	4
2	3	5
3	3	3
4	4	4
4	4	4
5	6	4

22. Now that data has been collected, analysis begins. These same observations are true:
    - all numbers involved are integers
    - there seems to be a relationship between the number of pins, the number of sticks, and the DOF
    - pins and sticks are independent variables, DOF is dependent

    - Again, two equations are chosen and used to solve for α and β.

Eq. 1: $3\alpha + 2\beta = 4$	→	multiply by -2	$-6\alpha - 4\beta = -8$
Eq. 2: $4\alpha + 4\beta = 4$	→	add to this eq.	$4\alpha + 4\beta = 4$
		yields	$-2\alpha = -4$, or $\alpha = 2$

Substituting in Eq. 1, we then find that $3(2) + 2\beta = 4$, or $\beta = -1$
This give the equation

$$2P - N = F$$

This formula, which differs from the first attempt, should now be tested by the students with their models. This activity should be completed and reported out by all groups of students.

Teachers' note: This completes level two. Please skip down to **Additional Proof – Alternative Approach** *that follows level three for an additional and useful activity.*

Level Three – *The Proof* - intended for advanced students

Level two established a seemingly verifiable equation to be used for determining DOF for a two dimensional structure. However, it can be seen that as structures become larger and more complicated it is either cumbersome or impossible to prove that the equation is true using the counting algorithm. Even if this is done, it is easy to imagine even larger and more complex structures.

Mathematicians and scientists require more definitive proof that such equations may be accurately applied to complex cases. Therefore, mathematical proofs must be offered before such equations become generally accepted. This section will lead to such a proof and give students an experience with a real physical case that has a mathematical proof.

1. Have students discuss what would be required to prove that the equation will work for all values of P and N. It is possible that no working ideas will be reached, but students should discuss the attributes that such a proof would have – that the equation be reached through actual mathematical manipulation of observed behavior.

2. A good approach would be to write out the counting algorithm (which works) and represent it mathematically. This will require some terminology to be established. In the counting algorithm each joint is evaluated to determine how many sticks meet there. Then a number of DOF is deducted based on the number of sticks at that joint. For example, if two sticks meet, then two DOF are deducted. This can be referred to as "loss of degrees of freedom" (LDF). If four sticks meet, LDF is six. Of course, in complex examples, there may be more than one joint of each type. We can represent these joints as

   J_r where r is the number of sticks in a joint.

   Consider the example of the square. There are four joints, each having two sticks. Thus for the square, $J_2 = 4$. We know that for each of these two-stick joints, LDF is two.

3. Students should now be asked to work in their groups to find a pattern among the different J_r values. The goal is to be able to write a mathematical expression for each J_r that will yield the total LDF for each J_r.

Teachers' note: The following is an example of how this might be done by students.

First, make a table for the different values of r and LDF:

r	LDF
1	0
2	2
3	4
4	6
5	8
6	10

The students should be able to analyze this and determine that the LDF is 2(r-1). The next step will be to use this to write an equation for the counting algorithm. First, it can be seen that:

$$F = 3N - 0J_1 - 2J_2 - 4J_3 - 6J_4 - \cdots$$

This is not a proper equation, since there is no way to express every possible J_r. Since this is an infinite series, it should be written in series notation. Furthermore, students should see that it will be more precise to use the expression for LDF found above. Doing that first we see:

$$F = 3N - 2(1-1)J_1 - 2(2-1)J_2 - 2(3-1)J_3 - 2(4-1)J_4 \ldots$$

Each of the subtracted terms is in the form $2(r-1)J_r$, so we can factor out the 2 from each term

$$F = 3N - 2[(1-1)J_1 + (2-1)J_2 + (3-1)J_3 + (4-1)J_{4\ldots}]$$

Then, using summation notation we can write this as

$$F = 3N - 2\sum_{r=1}^{r_{max}}(r-1)J_r$$

This part, $-2\sum_{r=1}^{r_{max}}(r-1)J_r$, represents the LDF in the equation and has something to do with the number of sticks at each pin.

Each step should be created by the students to the greatest extent possible. As always, the teacher must decide when to plant "clues" or make minimal suggestions to keep the work moving forward.

Now we see that the above expression shows pins in a form that includes all possible types of joints. We need a similar expression for the number of sticks in order to complete the proof. Let's go back to the "house" example. There are five pins and six sticks. The joints consists of three joints with two sticks and two joints with three sticks. But each stick has a joint at each end, so each joint represents half of each stick that meets there. So the three joints with two sticks have a total of 6 half-sticks and the two joints with three sticks have a total of 6 half-sticks. Added together, this gives us 12 half-sticks, or 6 sticks. This can be written mathematically as:

$$3\left(\frac{2}{2}\right)+2\left(\frac{3}{2}\right)=6$$

Note that the numerator of each fraction is in fact the *r* value, and the coefficient is J_r. This can be written in general as:

$$J_r\frac{r}{2}$$

and we can sum all the different joints to get the total number of sticks:

$$N=\sum_{r=1}^{r_{max}}J_r\frac{r}{2}$$

We can now substitute and write the equation for DOF:

$$F=3\sum_{r=1}^{r_{max}}J_r\frac{r}{2}-2\sum_{r=1}^{r_{max}}(r-1)J_r$$

Now everything, sticks and pins, is expressed in terms of J_r and *r*. If we can simplify this equation and re-express it in terms of sticks and pins, we will have proven our equation. We should now determine how to express the number of pins in terms of J_r and *r*. This is much simpler – for every joint, there is a pin. Thus if we add up the number of joints, we will know the number of pins:

$$P=\sum_{r=1}^{r_{max}}J_r$$

We can expand the second term of the DOF equation:

$$F=3\sum_{r=1}^{r_{max}}J_r\frac{r}{2}-2\sum_{r=1}^{r_{max}}[rJ_r-J_r]$$

We can now separate that second term:

$$F=3\sum_{r=1}^{r_{max}}J_r\frac{r}{2}-2\sum_{r=1}^{r_{max}}rJ_r+2\sum_{r=1}^{r_{max}}J_r$$

Now the second term of this equation can be changed by moving a constant multiplier of ½ into the summation and adding a factor of two to the coefficient:

$$F = 3\sum_{r=1}^{r_{max}} J_r\frac{r}{2} - 4\sum_{r=1}^{r_{max}} J_r\frac{r}{2} + 2\sum_{r=1}^{r_{max}} J_r$$

Since the first two terms are alike, they can be combined:

$$F = -1\sum_{r=1}^{r_{max}} J_r\frac{r}{2} + 2\sum_{r=1}^{r_{max}} J_r$$

Note that the first term is really the number of sticks, and the second term is the number of pins as determined above. We can now write this as

$F = -N + 2P$, which can be rearranged as $F = 2P - N$

This completes the proof of our equation.

Additional Proof – Alternative Approach

Throughout this unit, we have approached development in a stick-centric way. Students will do this naturally, but it turns out that a completely different viewpoint will provide a very simple verification of our work. This additional approach would also be appropriate for those students who complete the second level of this activity.

Suppose that instead of looking at sticks, we look at pins. We have already established that every hole must have a pin, why not look at it from the other direction? Let's consider a pin alone on the 2D plane. Since a pin represents a single point, it would have two DOF – the x and y directions. There is no rotation, since the pins are single points. Therefore two pins would have four DOF, three pins would have six DOF, etc.

If we add a stick to connect two pins, it can be observed that we now force the two pins to always be a specific distance apart. This can be called a "constraint". The two pins can still be moved, but wherever one is moved the other must always remain at that fixed distance. Thus two pins connected by a stick have lost a DOF, and thus two pins connected by one stick have three DOF. Remember that each time we add a stick to our system, every hole must have a pin.

Example: Suppose we have three pins on the plane. Together the pins would have six DOF. If we use three sticks to join the pins into a triangle, we would have those six DOF but would also have three constraints, so there would only be three DOF. This is consistent with our earlier study of the triangle. If we added a stick to one of the vertices, we must also add another pin to satisfy our requirement that every hole have a pin. There would then be four pins and four sticks, yielding:

$$2(4) - 4(1) = 4 \text{ DOF}$$

Have students use this information to either directly write a general equation or use tables and sample structures as before. Students should determine that:

$$F = 2P - N \text{ as before.}$$

Appendix 1 – State and National Standards

National Mathematics Standards

Algebra

In grades 9–12 all students should–

- generalize patterns using explicitly defined and recursively defined functions;

Represent and analyze mathematical situations and structures using algebraic symbols
In grades 9–12 all students should–

- judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Use mathematical models to represent and understand quantitative relationships

In grades 9–12 all students should–

- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;
- draw reasonable conclusions about a situation being modeled.

Apply transformations and use symmetry to analyze mathematical situations

In grades 9–12 all students should–

- understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices;
- use various representations to help understand the effects of simple transformations and their compositions.

Use visualization, spatial reasoning, and geometric modeling to solve problems

In grades 9–12 all students should–

- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;
- visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections;
- use vertex-edge graphs to model and solve problems;
- use geometric models to gain insights into, and answer questions in, other areas of mathematics;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Problem Solving

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Communication

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

Arizona Mathematics Standards

Strand 3: Patterns, Algebra, and Functions

Concept 3: Algebraic Representations

PO 4. Translate a written expression or sentence into a mathematical expression or sentence.
PO 5. Translate a sentence written in context into an algebraic equation involving multiple operations.
PO 8. Solve linear (first degree) equations in one variable (may include absolute value).

Strand 4: Geometry and Measurement

Concept 2: Transformation of Shapes

PO 1. Sketch the planar figure that is the result of two or more transformations.
PO 2. Identify the properties of the planar figure that is the result of two or more transformations.
PO 5. Classify transformations based on whether they produce congruent or similar figures.

Strand 5: Structure and Logic

Concept 1: Algorithms and Algorithmic Thinking

PO 1. Determine whether a given procedure for simplifying an expression is valid.
PO 2. Determine whether a given procedure for solving an equation is valid.
PO 4. Select an algorithm that explains a particular mathematical process.
PO 5. Determine the purpose of a simple mathematical algorithm.
PO 6. Determine whether given simple mathematical algorithms are equivalent.

Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

PO 3. Write an appropriate conjecture given a certain set of circumstances.
PO 4. Analyze assertions related to a contextual situation by using principles of logic.
PO 5. Identify a valid conjecture using inductive reasoning.
PO 6. Distinguish valid arguments from invalid arguments.
PO 9. Identify a counterexample for a given conjecture.
PO 10. Construct a counterexample to show that a given conjecture is false.

Arizona Science Standards

Strand 1: Inquiry Process

Concept 1: Observations, Questions, and Hypotheses

PO 2. Develop questions from observations that transition into testable hypotheses
PO 3. Formulate a testable hypothesis.
PO 4. Predict the outcome of an investigation based on prior evidence, probability, and/or modeling (not guessing or inferring).

Concept 2: Scientific Testing (Investigating and Modeling)

PO 5. Record observations, notes, sketches, questions, and ideas using tools such as journals, charts, graphs, and computers.

Concept 4: Communication

PO 3. Communicate results clearly and logically.
PO 4. Support conclusions with logical scientific arguments.

Strand 2: History and Nature of Science

Concept 1: History of Science as a Human Endeavor

PO 1. Describe how human curiosity and needs have influenced science, impacting the quality of life worldwide
PO 2. Describe how diverse people and/or cultures, past and present, have made important contributions to scientific innovations.
PO 3. Analyze how specific changes in science have affected society.
PO 4. Analyze how specific cultural and/or societal issues promote or hinder scientific advancements.

Concept 2: Nature of Scientific Knowledge

PO 1. Specify the requirements of a valid, scientific explanation (theory), including that it be:

- logical
- subject to peer review
- public
- respectful of rules of evidence

PO 2. Explain the process by which accepted ideas are challenged or extended by scientific innovation.
PO 3. Distinguish between pure and applied science.
PO 4. Describe how scientists continue to investigate and critically analyze aspects of theories.

National Science Standards

Note: The Arizona Science Standards are closely aligned with National Standards, and little would be served to repeat those standards here. For more details and for comparison with the Arizona standards we have identified for this unit of study, please see the National Science Standards at http://www.nap.edu/html/nses/ .